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Journal of Arid Environments 66 (2006) 657–672

Journal of Arid Environments

www.elsevier.com/locate/jnlabr/yjare

Wind tunnel modeling and measurements of the flux of wind-blown sand

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Received 2 January 2003; received in revised form 1 April 2005; accepted 21 November 2005
Available online 5 January 2006

Abstract

This paper presents a way to empirically fit experimental data for the horizontal flux of various sizes of wind-blown dry sand using data from wind tunnel experiments. We measured vertical wind profiles to derive threshold shear velocity and estimated shear velocity and the flux of sand mass as a function of the height for nine different grain sizes. We propose a fitting model based on the experimental data and a least-squares method and derive an explicit form of sand flux as a function of height and shear velocity for these grain sizes. We also obtained an explicit form of the empirical equation for the measurement of sand transport per unit width and unit time by integrating the empirical equation as a function of height. Finally, we compared the effectiveness of Bagnold's equation, Kawamura's expression and Lettau and Lettau's equation, for predicting sand transport with the results of our empirical equation. The results show that the transport predicted by all of the equations were always lower than the measured results from the empirical equation for all grain sizes and shear velocities. However, the empirical equation matched Bagnold's equation, Kawamura's equation, and Lettau and Lettau's equation if the coefficients in these equations were adjusted instead of using their original coefficients. The empirical equation for sand transport in the present

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0140-1963/$ - see front matter
doi:10.1016/j.jaridenv.2005.11.010

study contradicts previous conclusions generated by Bagnold's equation, which predict that for a given wind drag, the transport of a coarse sand is greater than that of a fine sand.

Keywords: Sand flux profile; Threshold shear velocity; Shear velocity; Grain size; Aeolian transport model

1. Introduction

Wind erosion and desertification are serious environmental problems in arid and semi-arid regions (Wu, 1987). We must understand sediment transport systems before it becomes possible to take reasonable measures to protect soils from wind erosion and to prevent the area desert from expanding (Lancaster, 1995; Sherman et al., 1998). Studies of sand transport processes have assumed great importance in the practice of modern geomorphology (Lancaster, 1995; Sherman et al., 1998). When the physics of the sand transport process are adequately understood, this knowledge allows the development of theory-based models capable of predicting system responses over short and medium time-scales. For fluid-based transport systems, such prediction requires accurate specification of the characteristics of the flow field, the sediment, boundary conditions, and interactions between different aspects of the system (Sherman et al., 1998).

Many models have been developed to predict rates of aeolian sediment transport (Bagnold, 1941; Kawamura, 1951; Zingg, 1953; Kuhlman, 1958; Owen, 1964; Kadib, 1965; Hsu, 1971; Kind, 1976; Maegley, 1976; Radok, 1977; Lettau and Lettau, 1977; Nakashima, 1979; Takeuchi, 1980; Horikawa et al., 1984; Sarre, 1987; Sherman and Hotta, 1990; Werner, 1990; Sherman et al., 1998), among which Bagnold's equation (1941), Kawamura's equation (1951), and Lettau and Lettau's equation (1977) are generally considered to be efficient. These equations are now widely used (Sherman et al., 1998). Bagnold's equation (1941) is expressed as follows:

$$Q = C\sqrt{\frac{d}{D}}\frac{\rho}{g}u_*^3, \tag{1}$$

where Q is the rate aeolian sediment transport, in kg m^{-1} s^{-1}; d is the mean grain diameter, in mm; D is a reference grain diameter, equal to 0.25 mm; u_* is shear velocity, in m s^{-1}; ρ is air density, equal to 1.25 kg m^{-3}; g is gravitational acceleration, equal to 9.81 m s^{-2}; and C is a constant that ranges from 1.5 to 2.8 depending upon the nature of the grains being transported, and has a value of 1.5 for uniform sands. We used $C = 1.5$ in the present study.

Kawamura (1951) was the first to propose a model with an explicit term to describe the threshold shear velocity, u_{*t}:

$$Q = K\frac{\rho}{g}(u_* - u_{*t})(u_* + u_{*t})^2, \tag{2}$$

where u_{*t} is in m s^{-1}, and Kawamura suggested a value of $K = 2.78$. In the present study, K is taken as 2.78.

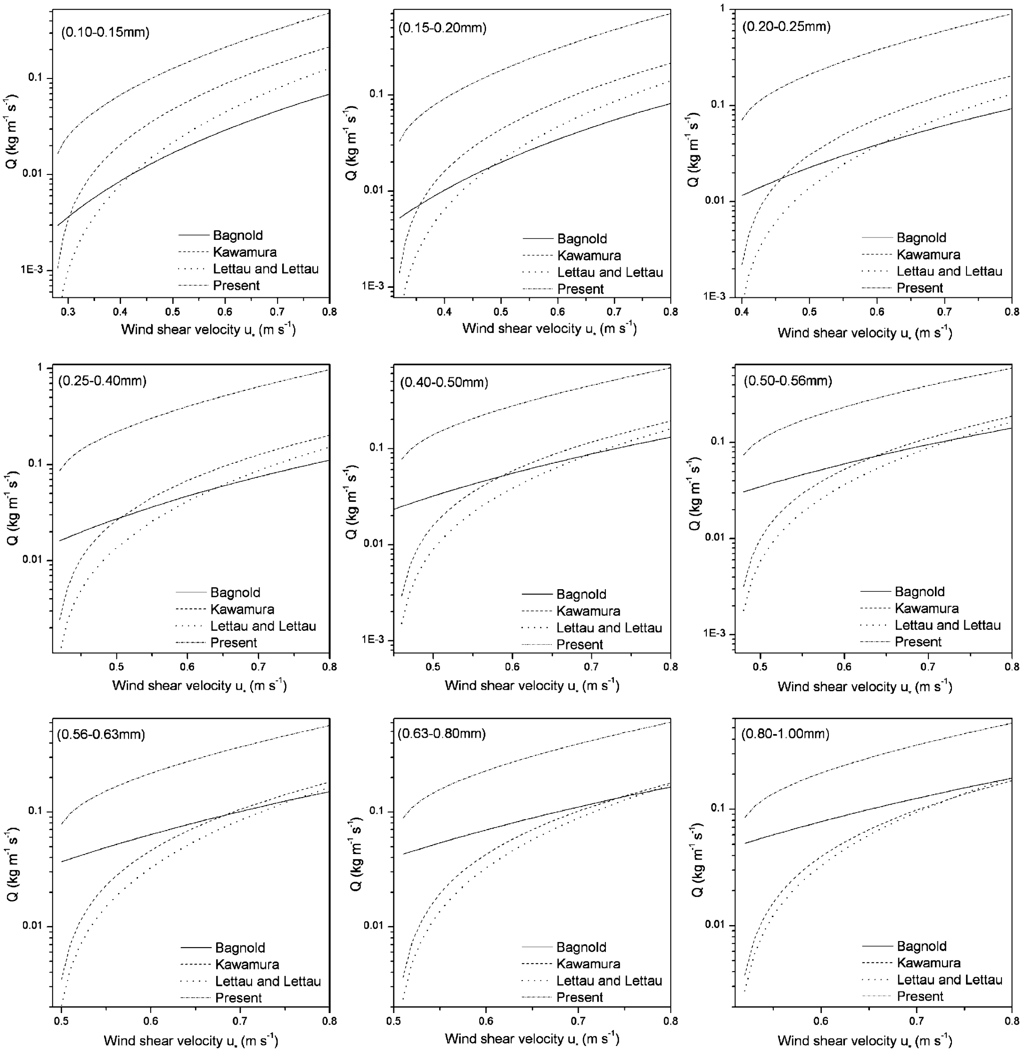

Fig. 1. Comparison of the measured results produced by Eq. (19) with the values predicted by Bagnold's (1941) equation, Kawamura's (1951) equation, and Lettau and Lettau's (1977) equation for the sand transport rate per unit width and unit time.

The model of Lettau and Lettau (1977) was derived through a series of experiments in arid environments. The formulation is

$$Q = L\sqrt{\frac{d}{D}}\frac{\rho}{g}(u_* - u_{*t})u_*^2, \tag{3}$$

where L has a value of 4.2.

Fig. 1 illustrates the response of the predicted rate of transport for sand grains with the different sizes employed in our experiment for these three models over a range of shear velocities between u_{*t} (the threshold shear velocity) and $0.8\,\mathrm{m\,s^{-1}}$. From Fig. 1, we can see that the rate of transport predicted by Kawamura's equation for all sand grain sizes is always greater than that of Lettau and Lettau's equation across the whole range of shear velocities. Bagnold's equation represents the upper limit for predicted rates of transport when shear velocities u_* are less than u_{*1}, but its predictions fall at the lower end of the spectrum when shear velocities exceed u_{*2}, and it provides an intermediate estimate of transport rates when $u_{*1} \leqslant u_* \leqslant u_{*2}$ as indicated in Fig. 1. Here, u_{*1} represents the point of intersection between the values predicted by Bagnold's equation and Kawamura's equation, and u_{*2} represents the point of intersection between the values predicted by Bagnold's equation and Lettau and Lettau's equation. Values of u_{*1} and u_{*2} for a range of grain sizes are listed in Table 1. From Table 1, it is clear that both u_{*1} and u_{*2} increase with increasing grain size. In addition, Fig. 1 suggests that the predictions of Bagnold's expression for sand with a 0.9 mm mean grain size is always greater than that predicted by Kawamura's equation for the whole range of shear velocities. By comparing the predictions of these three equations, it is clear that their predictions sometimes differ by a factor of 3 even though there is little difference in form among the equations (Sherman et al., 1998; Zhou et al., 2002).

Evaluating the accuracy of the equation depends mainly upon the precision of quality of the measured data for the rate of sand transport. Since this quality is not directly measurable, the precision of the experimental data is sensitively related to both the measured values of the mass flux of sand passing through a sand sampler and the method of processing of the measured data (Zhou et al., 2002). In practice, the processing of the observed data can be conducted by artificially extending the experimental data for the sand flux measured using a sand sampler; this analysis can be performed on graph paper to find an empirical curve, which can then be extended to the domain that lies beyond the height of the sand sampler, and the area between the curve and the axis of the graph can be calculated by means of numerical arithmetic. The area is then considered to represent the rate of sand transport (Liu, 1995). It is obvious that the value obtained for sand transport varies with the statistical empirical curve since it is not uniquely chosen. Until now, no general approach with a high precision and with a satisfactory equation or approach

Table 1
The values of u_{*1} and u_{*2} for different grain sizes (d) of sand[a]

d (mm)	Mean d (mm)	u_{*1} ($\mathrm{m\,s^{-1}}$)	u_{*2} ($\mathrm{m\,s^{-1}}$)
0.80–1.00	0.900	0.76	0.795
0.63–0.80	0.715	0.74	0.77
0.56–0.63	0.595	0.68	0.76
0.50–0.56	0.530	0.63	0.73
0.40–0.50	0.450	0.59	0.70
0.25–0.40	0.325	0.50	0.63
0.20–0.25	0.225	0.45	0.61
0.15–0.20	0.175	0.35	0.48
0.10–0.15	0.125	0.30	0.42

[a] u_{*1} represents the point of intersection of the prediction of the values predicted by Bagnold's (1941) equation and Kawamura's (1951) equation, and u_{*2} represents the point of intersection between Bagnold's equation and Lettau and Lettau's (1977) equation.

capable of reproducing the observed experimental data has been found. In particular, no explicit form of the empirical equation for the distribution of the mass flux of sand as a function of height and shear velocity has been found (Zhou et al., 2002). In this paper, we propose an approach based on an empirical equation for the mass flux of sand and sand transport rate for nine sizes of sand grains, based on experiments conducted in a wind tunnel. By means of the empirical equation obtained for the measurement of sand transport rate, we evaluated the relative accuracy of Bagnold's (1941) equation, Kawamura's formula and Lettau and Lettau's model when they are employed to predict the transport rate for differently sized sand grains tested in our experiment.

1.1. Experimental apparatus and results

The experiment was carried out in the wind tunnel at the Shapotou Desert Experimental Research Station, Cold and Arid Regions Environmental and Engineering Research Institute, Chinese Academy of Sciences. Dong et al. (2002b) provide more details about the wind tunnel.

The main purpose of this experiment was to measure the threshold shear velocity, shear velocity, and mass flux of blown sand with different grain sizes at different free-stream wind velocities. Natural sand from the field was sieved into nine size groups before we conducted our tests: 0.10–0.15, 0.15–0.20, 0.20–0.25, 0.25–0.40, 0.40–0.50, 0.50–0.56, 0.56–0.63, 0.63–0.80 and 0.80–1.00 mm.

The initiation of sand movement was observed following a procedure that has been used by several previous researchers (e.g. Musick et al., 1996), in which strips of double-sided sticky tape were placed flat on the bed surface near the downwind end of the test tray to capture moving grains by means of adhesion. The initiation of sand movement was observed visually and the wind was considered to have reached the initiation threshold when more than five particles stuck to the tape. For each grain size, three different observers observed the wind initiation threshold three times so that we could estimate the mean value of the threshold shear velocity and so that errors in visual observation could be reduced. We derived the threshold shear velocity by means of the least-squares curve fitting of the vertical wind profiles at the initiation threshold. Wind profiles were measured using a wind profiler made by the Shaanxi Air Instrument Company (Dong et al., 2002a), which contains 10 Pitot-static tubes at different heights. We fitted the measured wind speeds at 10 heights (3, 6, 10, 15, 30, 60, 120, 250, 350, and 500 mm above the floor of the chamber) using the following equation:

$$u(z) = t + s\ln(z), \quad (4)$$

where $u(z)$ is the wind speed at height z, and t and s are regression constants. We obtained the threshold shear velocity u_{*t} using the following equation:

$$u_{*t} = k{*}s, \quad (5)$$

where k is Karman's constant, which equals 0.4. The measurement at a height of 500 mm was eliminated from our curve fitting when we found that it was not within the logarithmic regions of the curves. The measured threshold shear velocities of the sands that we tested are listed in Table 2.

The flux of blown sand at different heights was measured using a segmented sand sampler (Dong et al., 2002b). The sampler is 60 cm tall and is sectioned into 60 openings

Table 2
The measured threshold shear velocity for the tested sands

d (mm)	0.10–0.15	0.15–0.20	0.20–0.25	0.25–0.40	0.40–0.50	0.50–0.56	0.56–0.63	0.63–0.80	0.80–1.00
u_{*t} ($\mathrm{m\,s^{-1}}$)	0.27	0.31	0.39	0.41	0.45	0.47	0.49	0.5	0.51
R^2	0.99	1	0.98	0.98	1	0.98	0.99	0.99	0.98

d is the sand grain size (diameter), and u_{*t} is the threshold shear velocity.

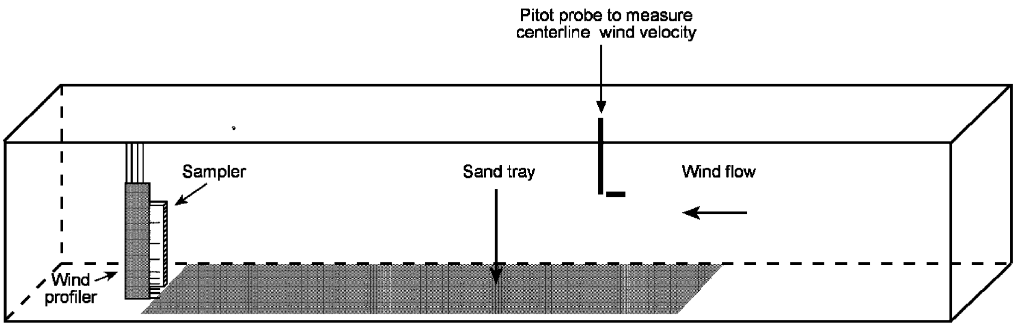

Fig. 2. The layout of the sand tray, sampler, and profiler in the wind tunnel.

(each 1 cm tall and 0.5 cm wide) to collect the blown sand at 60 heights at 1-cm intervals. Each opening is connected to a sand chamber that is removed after the test to weigh the sand that it has collected using an electronic balance with 0.001-g precision. The spacer between the openings has been made very thin to reduce the measurement error. To minimize the sampler's interference with the airflow, the leading edge of the sampler is funnel-shaped so that the width of the sand chamber is 1.5 cm while the width of the openings is only 0.5 cm. A screened vertical vent is connected to each sand chamber to minimize the air pressure in the sand chamber and maximize the collection efficiency. Evaluation of the wind tunnel using dune sand revealed that the sampler was effective at collecting sand larger than 0.1 mm in diameter that had insignificant interparticle cohesion and insignificant adhesion to the surface. The overall capture efficiency was greater than 90% (Dong et al., 2002b).

In our experiment, we positioned the sand sampler 16 m downwind from the entrance of the working section of the wind tunnel, at a location 0.1 m downwind from the sand tray that provided the source of windborne grains (Fig. 2). The bottom of the lowest opening of the sand sampler was set flush with the tunnel floor. The sand tray upwind of the sand sampler was 4 m long, 0.8 m wide and 0.025 m deep. We chose the length of the sand tray to ensure that significant development of the saltation cloud would occur. Sand samples were sieved (as described above) to ensure that they had a known range of particle sizes, and then were placed in the sand tray. We then set the sand surface to be level with the tunnel floor, and set the wind tunnel to generate the required wind velocity (measured at the centerline of the wind tunnel, 0.6 m above the tunnel floor), which was chosen to be above the threshold required for the initiation of saltation. Because it took some time for the wind tunnel to reach the preset wind speed, we covered the sand tray with an automatic sliding lid until the required wind speed was reached so as to prevent the sand from being blown away prematurely. For each sample at each wind velocity, we performed three replications to provide mean values. Fig. 2 illustrates the layout of the sand tray and sampler and profiler.

We also measured the wind profiles within the saltating cloud using the wind profiler (Dong et al., 2002a) that was positioned 50 mm from the sand sampler (Fig. 2). Once the movement began, the wind profile changed especially in layers nearer to the floor of the wind tunnel. The recorded wind profiles were in equilibrium with the saltating cloud. For example, Fig. 3 plots a set of profiles of the experimental wind velocity as a function of height using a semilogarithmic graph for sand with a 0.45 mm mean grain size.

Studies of aeolian sand transport should use appropriate values of shear velocity (u_*) values for specific wind regimes (Hsu, 1971). In the present paper, we used the following equation (Bagnold, 1941; Svasek and Terwindt, 1974; Li and Martz, 1995) to calculate appropriate u_* values for each profile that were adapted to the saltating sand grains:

$$u(z) = \frac{u_*}{k} \ln \frac{z}{z_0}, \tag{6}$$

where k is the Karman constant, which equals 0.4; $u(z)$ is the wind velocity at any height z, in m s^{-1}; and z_0 is the roughness height, in m. The measured wind speeds were fitted using

$$u(z) = f_1 + f_2 \ln(z), \tag{7}$$

where f_1 and f_2 are regression constants. We obtained the shear velocity using

$$u_* = kf_2 = 0.4f_2. \tag{8}$$

In shallow wind tunnels, the determination of u_* from the slopes of velocity profiles in a blowing sand cloud has been shown to be error-prone (Bagnold, 1941; Owen, 1964; Owen and Gillette, 1985; White and Mounla, 1991) when Coles' Law of the Wake is not applied. However, recent work (Spies et al., 1995) has demonstrated the uncertainty and difficulty associated with the use of Coles' Wake Law to correct u_* values in small wind tunnels. These authors recommended that the constant-stress region of the boundary layer should be kept large enough for the desired measurements when Coles' Wake Function will not be

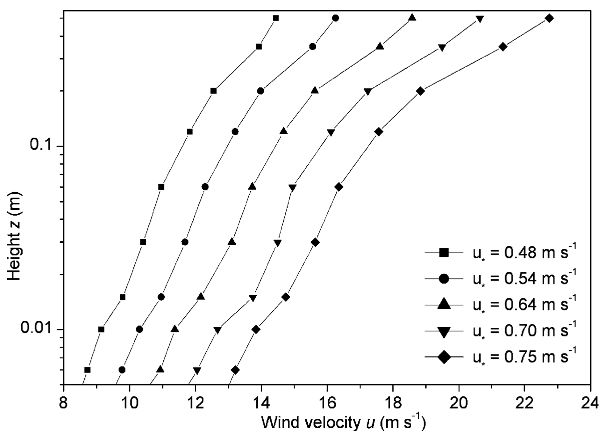

Fig. 3. The measured wind profiles within a cloud of saltating sand (grain size $d = 0.40-0.50$ mm).

used in the data analysis. In the present study, we used a wind tunnel that was 21 m long, 1.2 m wide, and 1.2 m high (Dong et al., 2002a) and the chosen length of the sand tray (4 m) ensured full development of the saltation cloud. The tunnel height of 1.2 m was adequate for the equilibrium boundary-layer flow, and was thus suitable for the determination of u_* from the slopes of the airflow velocity profiles (Nalpanis et al., 1993; Spies et al., 1995).

2. Fitting equation for the flux profile of a blowing sand cloud

2.1. Least-squares method of fitting sand flux curves

In this paper, we have used the least-squares method (LSM; Marquardt, 1963) to fit the experimental results. Let the equation to be fitted to experimental data be

$$y = y(x, \beta_1, \beta_2, \ldots, \beta_m), \tag{9}$$

where β_j $(j = 1, 2, \ldots, m)$ are the parameters to be determined. Assume that the experimental values of (x, y) are represented by (x_i, y_i) $(i = 1, 2, \ldots, n)$. From the LSM, we know that the parameters β_j $(j = 1, 2, \ldots, m)$ can be obtained by minimizing the following function:

$$\varepsilon = \sum_{i=1}^{n} \left[y_i - y(x_i, \beta_1, \beta_2, \ldots, \beta_m) \right]^2. \tag{10}$$

Thus, the parameters β_j $(j = 1, 2, \ldots, m)$ satisfy the following system of nonlinear algebraic equations:

$$\frac{\partial \varepsilon}{\partial \beta_j} = 0 \quad (j = 1, 2, \ldots, m). \tag{11}$$

When the fitting function of Eq. (9) degenerates into a linear function, we can take the superposition approach using preselected base functions; that is, we have $y = \sum_{l=j-1}^{m} \beta_l \delta_l(x)$. Here, $\delta_l(x)$ $(l = 1, 2, \ldots, m)$ is one set of known base functions. In this case, the nonlinear algebraic equations of Eq. (11) for the parameters β_j $(j = 1, 2, \ldots, m)$ become a set of linear equations. After the algebraic equations with unknowns are solved, the fitting equation is obtained.

2.2. Fitting equation for the flux of blown sand

Fig. 4 illustrates five sets of experimental results for the horizontal flux per unit width (1 m) per unit time (1 s) of sand with a grain size of 0.40–0.50 mm as a function of height for five wind velocities (marked by dots). From the measured data, we find that the values of the transport rate decreases with increasing height based on a negative exponential function for all the grain sizes and wind velocities that we tested. In this case, we used the following exponential equation as a best guess to fit the experimental data, as was done by Zhou et al. (2002):

$$q(z) = C + A \exp\left(-\frac{z - z_0}{B}\right), \tag{12}$$

where z_0 is the offset of z, C is the bias of $q(z)$, A is the amplitude, and B is the decay constant with respect to z. From the LSM, z_0 is taken as an appropriate fixed number that

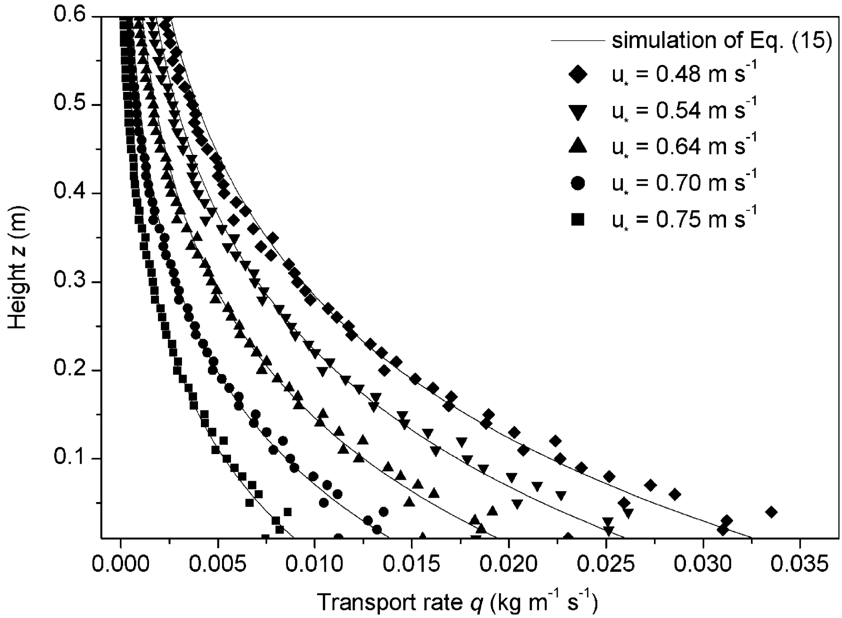

Fig. 4. Comparison between the fitting curves and the experimental data for sand flux (grain size, $d = 0.40-0.50\,\mathrm{mm}$ sand).

is close to the minimum value of the variable z. The bias C is set to be a fixed number close to the asymptotic value for the function $q(z)$ when z is sufficiently large. For the experiments in this paper, z_0 was taken as the position of the height of the lowest opening in the sand sampler, i.e. $z_0 = 0$. It is evident that when z becomes sufficiently large as the height increases, $q(z)$ will approach zero if the effect of grain suspension is neglected. Thus, we have $C = 0$. We can then obtain the values of the coefficients A and B in Eq. (12) using the LSM for the measured values of flux of blown sand for all grain sizes and wind velocities that we tested, which are indicated in Fig. 5 by dots and are referred to as the measured data for the coefficients. It is obvious from Fig. 5 that the values of A and B change with the shear velocity; that is, $A = A(u_*)$ and $B = B(u_*)$.

2.3. Fitting function for $A = A(u_*)$ and $B = B(u_*)$

The key step in finding suitable fitting equations for $A = A(u_*)$ and $B = B(u_*)$ in Eq. (12) is to develop an explicit form of the empirical equation for the flux of blown sand in the experiment, and another suitable fitting equation for transport rate as a function of shear velocity, as discussed later in this paper. When $C = 0$ in Eq. (12), we know that the coefficient A represents the intensity of the sand flux. In order to account for the fact that sand movement occurs only when the wind's shear velocity exceeds the threshold value (according to the experimental data for A in Fig. 5), we found a satisfactory test function after many attempts:

$$A = A(u_*) = a_1^*(1 - R_t)^{0.5} u_*, \tag{13}$$

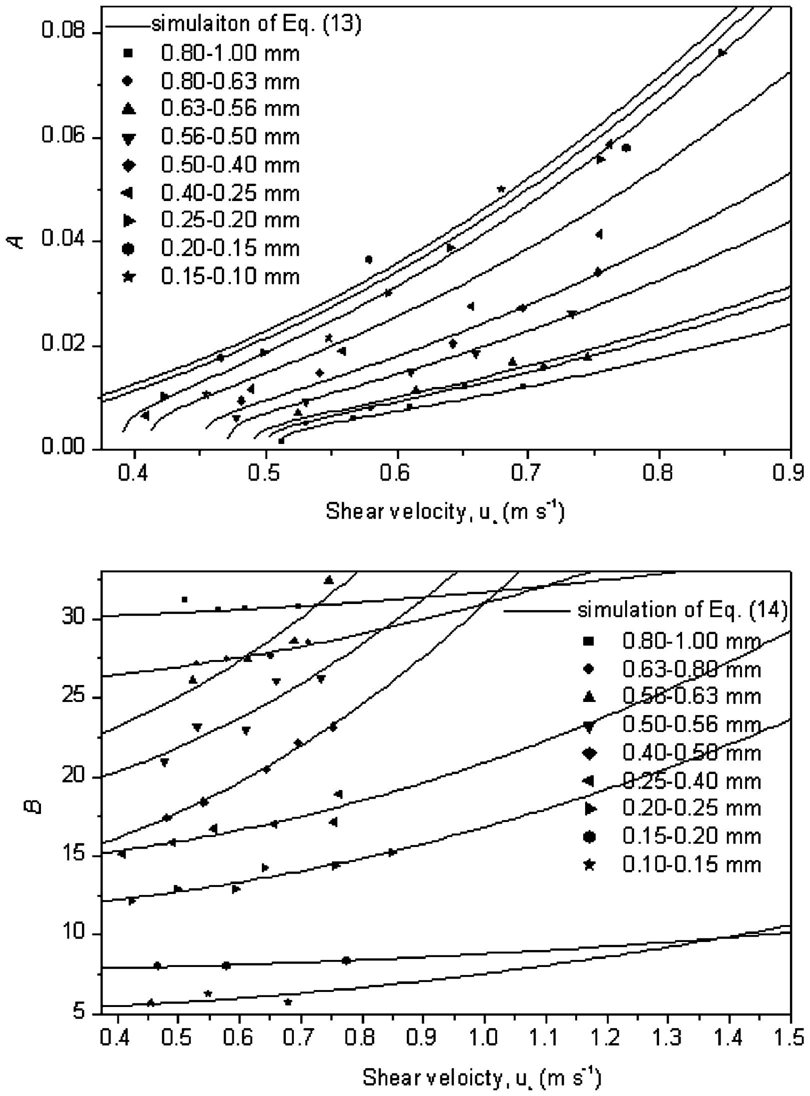

Fig. 5. Comparison of the experimental data for the coefficients A and B in Eq. (12) with the predictions of empirical equations (13) and (14).

where R_t is the ratio of the threshold shear velocity u_{*t} to shear velocity u_*, and A is in $m\,s^{-1}$. Here, a_1 is the dimensionless coefficient that must be determined. For the nine sand-size groups used in our experiment, we used Eq. (13) to generate values for coefficients a_1 (Table 3) by applying the LSM to the test results for u_* and u_{*t} that were introduced in the previous section (Fig. 5).

Similarly, for $B = B(u_*)$, we used the following second-order polynomial function:

$$B = B(u_*) = b_1 + b_2 u_*^2, \tag{14}$$

Table 3
Correlations between shear velocity (u_*) and efficients for A and B in Eqs. (13) and (14)[a]

d (mm)	a_1	R^2	b_1	b_2	R^2
0.80–1.00	0.0329	0.99	29.94	1.76	0.84
0.63–0.80	0.0394	0.99	25.62	5.341	0.90
0.56–0.63	0.0425	0.95	19.79	20.99	0.88
0.50–0.56	0.0555	0.94	17.65	16.8	0.84
0.40–0.50	0.0667	0.96	13.31	17.66	0.99
0.25–0.40	0.0735	0.93	14.24	6.662	0.81
0.20–0.25	0.1069	0.93	11.37	5.441	0.90
0.15–0.20	0.0917	0.93	7.74	1.069	0.93
0.10–0.15	0.0736	0.92	5.119	2.44	0.91

[a]Fitting functions: $A = a_1(1 - u_{*t}/u_*)^{0.5}u_*$ and $B = B(u_*) = b_1 + b_2u_*^2$, u_* and u_{*t} are in $\mathrm{m\,s^{-1}}$, and R^2 is the correlation coefficient.

where B is in $\mathrm{m^2\,s^{-2}}$, b_1 is in $\mathrm{m^2\,s^{-2}}$ and b_2 is a dimensionless proportionality coefficient. The values of coefficients b_1 and b_2 that we obtained are listed in Table 3. Obviously, the experimental data depend on many properties of the sand sample, such as sand grain size and the physical and chemical properties of the sand. Thus, the fitting coefficients a_1, b_1 and b_2 will depend on the sand sample employed.

2.4. Fitting results

Fig. 5 compares the experimental data for A and B in Eq. (12) with the predicted values from Eqs. (13) and (14). We found that the results given by empirical Eqs. (13) and (14) agreed well with the experimental data for A and B as a function of wind shear velocity. Substituting the obtained Eqs. (13) and (14) into Eq. (12), we get an explicit form of the empirical equation $q(z)$ as a function of shear velocity and height z:

$$q(z) = A(u_*)\exp\left(-\frac{z}{B(u_*)}\right). \tag{15}$$

In Fig. 4, the solid curves indicate the predicted values of Eq. (15) and the dots represent the measured experimental data. Fig. 4 shows that the values predicted by Eq. (15) agree with the measured data reasonably well.

3. Empirical equation for sand transport rate

As pointed out in the introduction, it is important to know how to predict the experimental data of sand transport rate as accurately as possible on the basis of the inadequate measured data for the mass flux of sand. In this section, we provide an explicit form for the relevant empirical equation.

Integrating Eq. (15) with respect to the variable of height z produces an equation for sand transport rate per unit width (1 m) per unit time (1 s), denoted by Q, in the following form (Zingg, 1953; Owen, 1964):

$$Q = \int_0^\infty q(z)\,\mathrm{d}z. \tag{16}$$

Substituting Eq. (15) into Eq. (16), we obtain

$$Q = A(u_*)B(u_*) = a_1(1 - R_t)^{0.5}u_*(b_1 + b_2u_*^2)$$
$$= a_1b_1(1 - R_t)^{0.5}u_*\left(1 + \frac{b_2}{b_1}u_*^2\right). \quad (17)$$

The coefficient a_1b_1 is a function of grain size and is expressed by (Fig. 6) by the following equation:

$$g_1(d) = a_1b_1 = \frac{1}{12.23 - 134.06d + 584.97d^2 - 1183.67d^3 + 1126d^4 - 406.89d^5},$$
$$R^2 = 0.92, \quad (18)$$

where the average value of (b_1/b_2) is 0.57. Substituting Eq. (18) into Eq. (17) and making some dimensional treatment produces the following equation:

$$Q = g(d)(1 - R_t)^{0.5}\left(\frac{\rho}{g}\right)u_*(1 + 0.57u_*^2),$$
$$g(d) = \frac{1}{1.56 - 4.27\left(\frac{d}{D}\right) + 4.66\left(\frac{d}{D}\right)^2 - 2.36\left(\frac{d}{D}\right)^3 + 0.56\left(\frac{d}{D}\right)^4 - 0.05\left(\frac{d}{D}\right)^5},$$
$$R^2 = 0.92, \quad (19)$$

where Q is in kg m^{-1} s^{-1}, d is in mm, u_* and u_{*t} are in m s^{-1}, $R_t = u_{*t}/u_*$, ρ is 1.25 kg m^{-3}, g is 9.81 m s^{-2}, and D is the reference grain diameter (0.25 mm). By means of this approach, we have obtained an explicit form of the empirical equation to formulate the experimental data for sand transport rate, which exhibits the fact that $Q = 0$ when $u_* = u_{*t}$ according to Eq. (13).

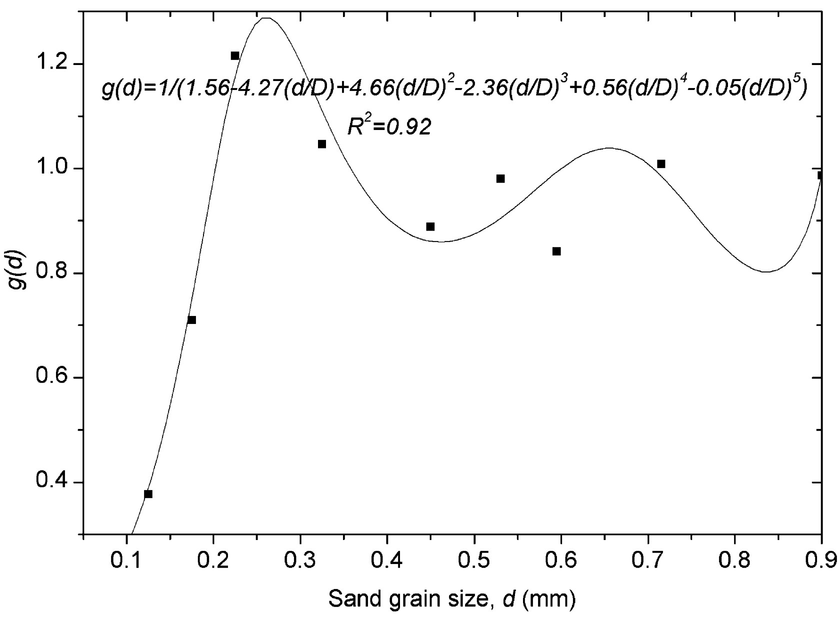

Fig. 6. Variation of $g(d)$ in Eq. (19) as a function of grain size.

In Eq. (19), $g(d)$ has a maximum at $d = 0.225$ mm, and increases with increasing grain size when $d < 0.225$ mm but decreases with increasing grain size when $d > 0.225$ mm, this finding contradicts Bagnold's (1941) conclusion in a qualitative sense, since Bagnold attributed the greater transport rate of coarser sand to the more intense impacts that occur during saltation. Based on the present study, it appears that the significance of these impacts may have been overestimated.

4. Results and discussion

In this paper, we compared three well-known equations (Bagnold, 1941; Kawamura, 1951; Lettau and Lettau, 1977), for predicting the sand transport rate per unit width and unit time. We compared their predictions with the measured results of Eq. (19) which we obtained in the present study for the nine sand-size groups that we tested.

Fig. 1 illustrates the response of the predictions of the rate of transport for the four equations over a range of shear velocities between the threshold shear velocity u_{*t} and $0.8\,\mathrm{m\,s^{-1}}$ for all of the sand samples that we tested. From Fig. 1, the transport rates predicted by Eq. (19) (the equation developed in the present study) were almost always larger than those predicted by Bagnold's equation (1), Kawamura's equation (2) and Lettau and Lettau's equation (3). The results of this comparison are presented in Table 4. We performed this comparison by calculating three ratios between the following parameters: the measured results given by Eq. (19) (Q_P), and the sand transport rates predicted by Bagnold's equation (Q_B), by Kawamura's equation (Q_K), and by Lettau and Lettau's equation (Q_L). These ratios are as follows:

$$r_B = Q_P/Q_B,$$
$$r_K = Q_P/Q_K,$$
$$r_L = Q_P/Q_L.$$

In Table 4, $C_B = 1.5r_B$, $C_K = 2.78r_K$ and $C_L = 4.2r_L$. From Table 4, it is clear that the values of the three ratios (i.e. r_B, r_K and r_L) vary with the sand grain size of the sand. Our results show that r_B ranges from 3.5 to 11.8, which disagrees with the conclusions of Sherman et al. (1998). They suggested that the ratio of the observed sand transport rates for sand with a 0.17-mm mean grain size on beaches to the predictions of Bagnold's

Table 4
The values of r_B, r_K, r_L, C_B, C_K and C_L, where "$r_B = Q_P/Q_B$", "$r_K = Q_P/Q_K$" and "$r_L = Q_P/Q_L$"; $C_B = 1.5r_B$, $C_K = 2.78r_K$ and $C_L = 4.2r_L$

d (mm)	Mean d (mm)	r_B	C_B	r_K	C_K	r_L	C_L
0.80–1.00	0.900	3.5	5.3	4.8	13.3	4.9	20.6
0.63–0.80	0.715	4.1	6.2	4.7	13.1	5.4	22.7
0.56–0.63	0.595	4.7	7.1	4.8	13.3	5.9	24.8
0.50–0.56	0.53	4.7	7.1	4.5	12.5	5.7	23.9
0.40–0.50	0.45	5.3	8	4.8	13.3	6.7	28.1
0.25–0.40	0.325	7.6	11	6.3	17.5	9.6	40.3
0.20–0.25	0.225	11.8	18	6.8	18.9	12	50.4
0.15–0.20	0.175	11.3	17	5.1	14.2	9.2	38.6
0.10–0.15	0.125	7.9	12	2.8	7.78	5.6	23.5

equation was 1.25, whereas the corresponding r_B in Table 4 was 11.3. This difference may have been caused by the different experimental conditions (e.g. sand moisture content, variable wind speeds). In addition, our values of r_K ranged from 2.8 to 6.8, and r_L ranged from 4.9 to 12.0 depending upon sand grain size. The performance of the equations, as represented by these three ratios, could be improved through manipulation of the empirical constants used in the equations. For example, the measured data given by Eq. (19) were best correlated with the predictions of Bagnold's equation if C in Eq. (1) equals C_B in Table 4, which varies as a function of grain size, rather than using a constant value of 1.5. Similar improvements can be obtained for the other equations. The predictions of Eq. (19) were best described by Kawamura's equation if K in Eq. (2) equals C_K in Table 4 instead of 2.78; for Lettau and Lettau's equation predictions improve if L in Eq. (3) equals C_L in Table 4 instead of 4.2.

5. Conclusions

In the present study, we developed a methodology for fitting experimental data for the flux of wind-blown dry sand of different sizes. Based on this approach, we derived an explicit form for the empirical equation that relates the sand flux as a function of height and shear velocity to the sand's saltation and creep motions. We then obtained an empirical equation for the observed results of the sand transport rate per unit width and unit time, which is a useful and important tool for modeling experimental data as accurately as possible. In the equation, transport rate increases with increasing grain size when mean grain diameter $d < 0.225$ mm but decreases with increasing grain size when $d > 0.225$ mm, with a maximum at $d = 0.225$ mm. The effect of grain size on sand transport rate disagrees qualitatively with the prediction of Bagnold's (1941) equation, which may overestimated the significance of impacts in the sand transport of larger sand grains.

On the basis of the equation developed in this paper, we evaluated the effectiveness of the published equations for predicting sand transport. For loose dry sand with a range of sizes, we found that the empirical equation obtained in this study for the experimental data gave results close to those of Bagnold's equation if C in Eq. (1) equals C_B in Table 4 (which varies with grain size) instead of a fixed value of 1.5; we obtained similar results for Kawamura's expression if K in Eq. (2) equals C_K in Table 4 instead of 2.78, and for Lettau and Lettau's (1977) equation if L in Eq. (3) equals C_L in Table 4 instead of 4.2. In addition, we had no difficulty in generalizing the fitting program displayed in this paper to other cases of wind-blown sand movement in which saltation is the dominant process.

Acknowledgements

We gratefully acknowledge the funding we received from the National Science fund for Distinguished Young Scholars of the Natural Science Foundation of China (40225003) and from the Natural Science Foundation of China (40301005, 40271114) and the Key Technologies Research and Development Program of the Tenth Five-year Plan of China (2002BA517A10). We thank Mr. Guochang Wang for his help in the wind tunnel experiment, and Dr. Guangjian Wu for preparing the illustrations.

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