A distance variable to simulate the urban population

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ABSTRACT

The urban population density declines from Central Business District (CBD) to city edge. Some studies have explored several mathematical forms to describe the relationship between population density and location. But these models assume that the city is single center and the urban population decline with concentric circle format. It is difficult to satisfy this requirement in actual cities. A distance variable, city edge distance, has been developed to solve the problem in this paper. Using city edge distance can express multi-centered city situation. The irregular city is easily transformed into a virtual concentric circle city. Then the Clark model and power exponential model are transformed into new format based on the city edge distance. A case study indicates that the transformed formats could successfully simulate the urban population density. But more applications are needed in different city types to validate and improve this distance variable.

Keywords: urban edgy distance; urban population; multi-centered city; irregular city; remote sensing and GIS

1. INTRODUCTION

Knowledge of the size and spatial distribution of human population in an urban area is essential for understanding and responding to myriad social, economic, and environmental problems such as emergency response and environmental impact assessment (Liu, 2003). Although many people have noticed the decrease of population density from inner city to outer, the study of urban population density must trace back to the classic study by Clark (Clark, 1951). Based on the convincing proof according to the statistic analysis of more than twenty western cities, Clark illustrated the negative exponential relationship between urban population and the distance from the city center (CBD) in the function:

$$D(r) = D(0)e^{br} \tag{1}$$

where r is the distance from one place in the city to the city center (CBD), D(r) is the population density at distance r from the CBD, D(0) is the density at distance zero, and b is the density gradient, D(0)>0 and b<0. This relationship has been demonstrated to exist for many cities of the United States, as well as for many cities outside the U.S. (Wu *et al.*, 2005; Weiss, 1961; Newling, 1965).

Some studies have explored other mathematical forms to describe the relationship between population density and location. For example, Sherratt (1960), Tanner (1961), and Sutton *et al.* (1997a) proposed a normal density model with a Gauss function:

$$D(r) = D(0)e^{br^2} \tag{2}$$

where b<0. Smeed (1963) gave the relationship:

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$$D(r) = D(0)r^{\mathrm{b}} \tag{3}$$

where $b<0$. Newling (1969) modified the work of previous researchers with two quadratic forms of exponential model, indicating a population density crater surrounding the central business district (Joan *et al.*, 2001). The logarithmic transformation of Newlig's modification produces a partial upside-down U-shaped curve:

$$D(r) = D(0)e^{br+cr^2} \tag{4}$$

The entropy-maximization methodology was used to derive Clark's model which assumes that population density D(r) at distance r from the center of the city (r=0) declines monotonically according to the negative exponential function, being based on the hypothesis of a circular city on ideal geo-surface (Chen, 2000a and 200b). Based on the idea of limited increasing entropy according to actual geographic condition, a new general model with power exponential format was improved:

$$D(r) = D(0)e^{br^{\sigma}} \tag{5}$$

which could integrate Clark model and Sherratt model (Chen, 1999). When σ =1, the formula (5)becomes Clark model; and When σ =2, it becomes Sherratt model. The deduction proved that concentric circle distribution and negative exponential decline were basic requirements. That is to say, the above models must assume that the city is single center and the urban population decline with concentric circle format.

But with the development at a high speed, most of the cities have irregular form and no longer distribute with concentric circle around the city center. Some cities do not have one center, but have two or more centers. The population simulation by the above models would distract from the actual situation. The research in the U.S.A. by Sutton (1997b) indicated that by measuring distance from the edge rather than the distance from the center this method allows for the "multiple nuclei" of urban clustering that have clearly manifested as a result of the conurbation of urban centers. Lu *et al.* (2002) used kernel estimation to model feasibly and effectively the spatial continuous surface of population density for multi-centered cities. In this paper, a distance variable, city edge distance, has been developed to solve the problem of un-concentric circle and multi-centered cities in simulating the urban population density.

2. METHODOLOGIES

City edge distance is the distance from one place in the city to the closest city edge. To compare with *r*, the city edge distance is expressed by r'. When $r'=0$, the place is the city edge line; and when r' reaches the maximum value (r'_{max}), the place is the city center, which may be not the real CBD and is just the theoretically calculated city center. The Ganzhou district is selected as a case study (Fig. 1). The city boundary was extracted based on the ASTER image in August 26, 2000.

Fig. 2 shows the city edge distance of Chengguan Town. The r'_{max} locates in the *a* place, which is the city center position. Moreover, three *b* places are the sub-centers of the city. Therefore, using city edge distance can express multi-centered city situation. Using city edge distance can also transform an irregular city to virtual concentric circle city (Fig. 2). The maximum radius of this virtual city is r'_{max}. The place of the irregular city when city edge distance equals r' would locate in the ring of the virtual city with the radius of $r'_{max}- r'$. In this way, the irregular city is easily transformed into a concentric circle city, which would fit into the basic requirement of the above model. All the outermost edges of the city have the zero city edge distance, so they locate in the outermost ring in the virtual concentric circle. Using city edge distance need assume that the city's outermost edges have the same population density. Because the city edge is normally the transitional zone between urban area and rural areas, the population density of city edge could set as the average population density of rural residential areas.

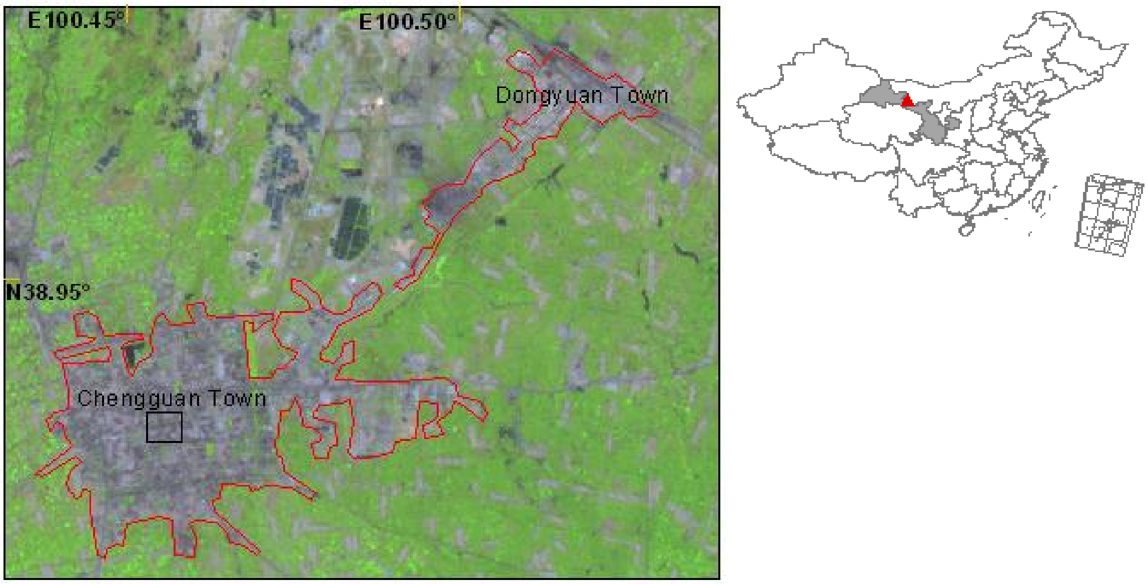

Fig. 1. Ganzhou district (right), position of Ganzhou in China (up-left), and city center (down-left). The image in right map is ASTER (Advanced Spaceborne Thermal Emission and Reflection Radiometer) image in August 26, 2000.

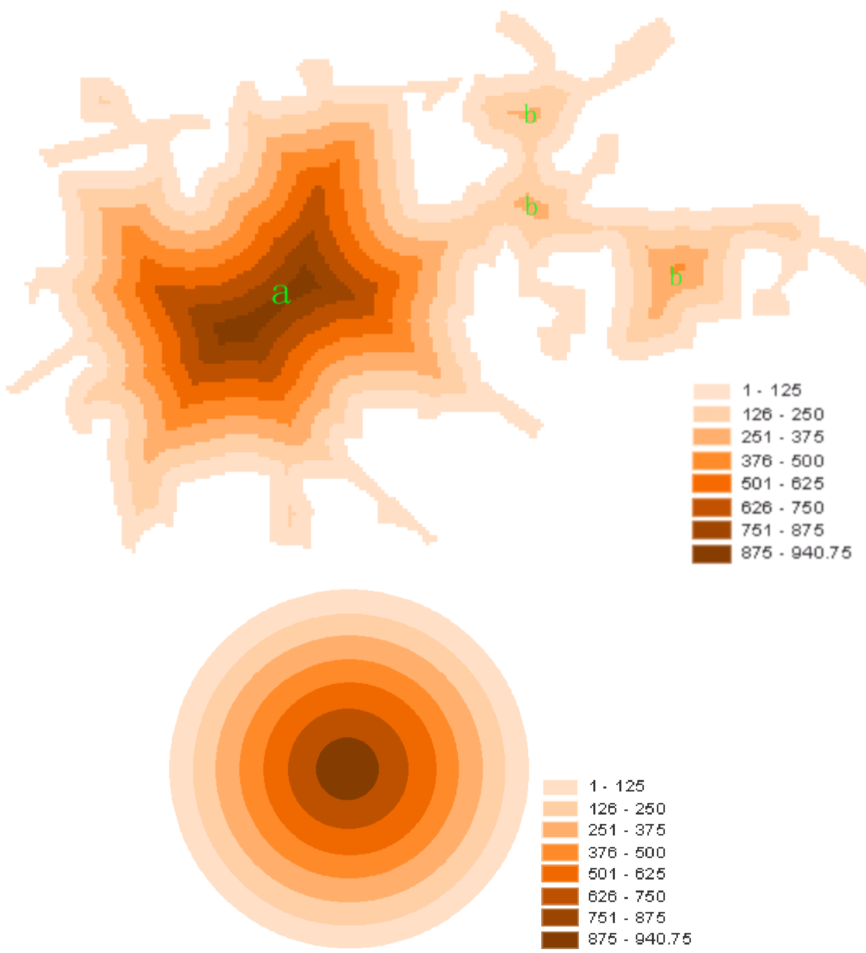

Fig.2. City edge distance of Chengguan Town, Ganzhou district (up) and virtual concentric circle city (down, unit: m)

When $D'(r')$ is assigned as the population density at distance r' from city edge. When city edge distance is r', $r = r'_{max} - r'$. The same place would have same population density, so:

$$D'(r') = D(r'_{\max} - r') \tag{6}$$

According to the Clark model and (6):

$$D(r'_{\max} - r') = D(0)e^{b(r'_{\max} - r')} = D(0)e^{br'_{\max}}e^{-br'} \tag{7}$$

When $r = r'_{max}$, the place is the ragged edge of the virtual concentric circle, $r'=0$. So:

$$D(r'_{\max}) = D(0)e^{br'_{\max}} = D'(0) \tag{8}$$

According to (6), (7) and (8):

$$D'(r') = D'(0)e^{-br'} \tag{9}$$

Fig.3 shows that the Clark model based on the distance from CBD, the population density declines from CBD. On the contrary, the population ascends from city edge to city center when the Clark model is based on city edge distance (Fig. 3).

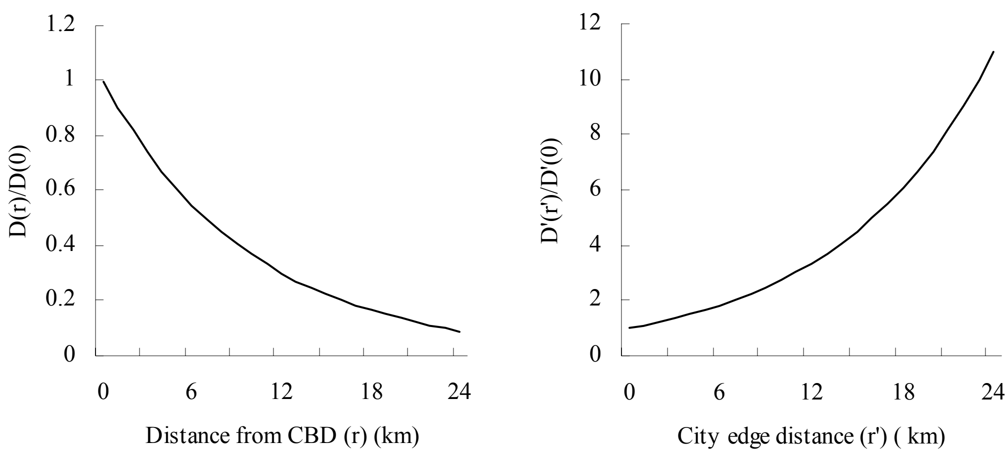

Fig. 3. Clark ‘s population density change based on distance from CBD (left) and city edge distance (right)

For power exponential model (5):

$$D'(r') = D(r'_{\max} - r') = D(0)e^{b(r'_{\max} - r')^{\sigma}} \tag{10}$$

$$D(r'_{\max}) = D(0)e^{b(r'_{\max})^{\sigma}} = D'(0) \tag{11}$$

According to (10) and (11), the power exponential model is transformed based on city edge distance:

$$D'(r') = D'(0)\frac{e^{b(r'_{\max}-r')^{\sigma}}}{e^{b(r'_{\max})^{\sigma}}} = D'(0)e^{b(r'_{\max}-r')^{\sigma}-b(r'_{\max})^{\sigma}} \tag{12}$$

3. CASE STUDY

The field survey and questionnaire indicate that the houses of the rural residential areas around Ganzhou district are gathered in village level and arranged in the shape of regular rectangles. The area of the rural residential areas has significant positive correlation with population. So the rural residential population density around Ganzhou district, $D'(0)$ = 3 999.18 person/km^2, was calculated according to census data and spatially rural residential area data derived from ASTER image in August 26, 2000 (Wang *et al.*, 2007).

In china, the cities are classified into four types according to the Non-agriculture Population in urban area. The Population of small-size city is less than 200 000 people. The Population of middle-size city is less than 500 000 people and greater than 200 000 people. The Population of big-size city is less than 1 000 000 people and greater than 500 000 people. The Population of metropolis is greater than 1 000 000 people. Both the populations of Chengguan Town and Dongyuan Town in Ganzhou district are less than 200 000 people and below to small-size cities.

Because there is no detail statistics data of the regional population in Ganzhou district, the population's spatial distribution status was investigated based on the floor number of the buildings derived from Quickbird image in March 22, 2004. The buildings in city edge are mostly one floor, which is just same as the rural residential area and the regional range is very narrow. The following buildings are two floors and the first floor is commercial usage and the second floor is inhabited usage. Its regional range is also narrow. Then the following buildings are three to five floors. The first and second floors are commercial usage and the other floors are inhabited usage. Its regional range is very broad. Lastly the CBD buildings are mostly five or six floors. The first and second floors are commercial usage and the other floors are inhabited usage. Its regional range is broad. A plausible character could be indicated for the population's spatial distribution status of Ganzhou district, a small-size city in Northwestern China. The population density quickly increases at city edge and slowly increases when close to city center.

If the city is single center and concentric circle distribution, when the total population, population density of city center and city edge are obtained, the b and σ could be calculated for the Clark model and power exponential model. But the city shape is irregular. Even though the city is transformed into a virtual concentric circle, the increasing area when Δr is not $\pi(\Delta r)^2$ (normally greater than $\pi(\Delta r)^2$). It is difficult to calculate the b and σ values. Therefore the parameters are calculated by adjusting parameters to infinitely access the actual total population number.

Clark model (9) has only a parameter b, which can be calculated according to the total population and population density of city edge. But the simulated $D'(r_{max})/D'(0)$ maybe have big deviation with actual situation. For example, the $D'(r_{max})/D'(0)$ is 19.8 and the population density of city center reaches 79 184 person//km^2 in Chengguan Town (Table 1). This value is higher than the CBD's population density of most of the special big cities in the world (Ding, 2004). It is also higher than the CBD's population density of Hangzhou City (28 184 person /km^2) and Chongqing City (30 000 person /km^2) in 2000 (Feng, 2002; Zhang *et al.*, 2003). There is different increasing trend of population density from city edge to center between simulated and actual situation. Therefore the transformed Clark model cannot effectively simulate the population density of Ganzhou district.

Because the population density of city center is unknown, it is unable to calculate the parameters *b* and σ of power exponential model just based on the total population and population density of city edge. By referring to the other Chinese cities CBD population density, the $D'(r_{max})/D'(0)$ is set as less than 6 for Ganzhou district. Then the b and σ are calculated and the increasing trend is obtained (Table 1 and Fig. 4). Obviously, it is much similar to the investigated situation of the spatial population. Lastly the power exponential model is used to simulate the urban population density of Ganzhou district (Fig. 5).

4. CONCLUSIONS

The city edge distance can effectively solve the problems of multi-centered and un-concentric circle cities in simulating urban population density. The Classical models of urban population density, such as the Clark model and power exponential model, can be transformed into new formats based on the city edge distance. In the case study, the power exponential model is suitable to simulate the urban population density of Ganzhou district. But some uncertain factors exist and need more applications of different type cities to validate and improve this distance variable. It is important to confirm the city edge lines. But how to obtain high-accuracy city edge would be a challenge in real operation. The simulated city center or sub-centers may not be the real centers. How to calculate the population density of the city edge? We assume the population density of the outermost city edge is same. But it is not so in the real world. How to solve it? We strongly hope others to try to use the city edge distance in different type cities and improve it especially when they have good spatial census data and can validate the simulated result in detail.

Table 1. Simulated parameters of Clark model and power exponential model for Ganzhou district

Town name	Population (person)	r_{max} (km)	Clark model		Power exponential model		
			b	$D'(r'_{max})/D'(0)$	*b*	σ	$D'(r'_{max})/D'(0)$
Chengguan Town	105793	0.941	-3.1738	19.8	2.2234	4	5.7
Dongyuan Town	13414	0.288	-7.2614	8.1	28.181	2.5	3.5

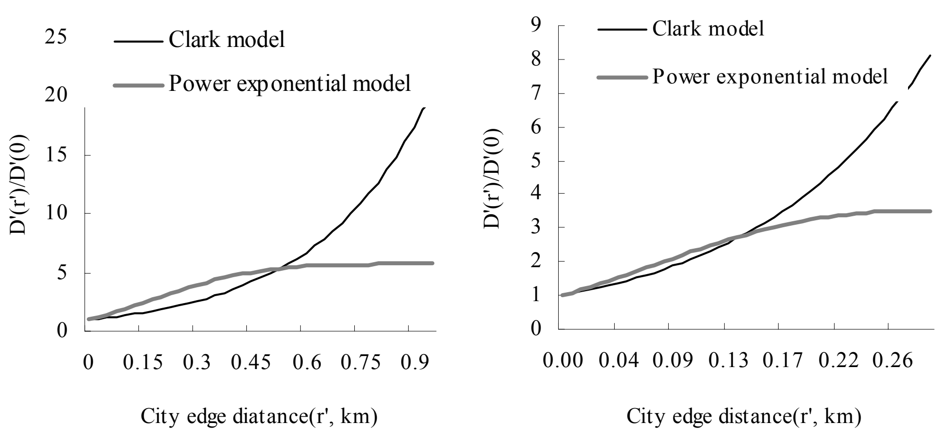

Fig. 4. Population's spatial variation with city edge distance in Chengguan Town (left) and Dongyuan Town (right)

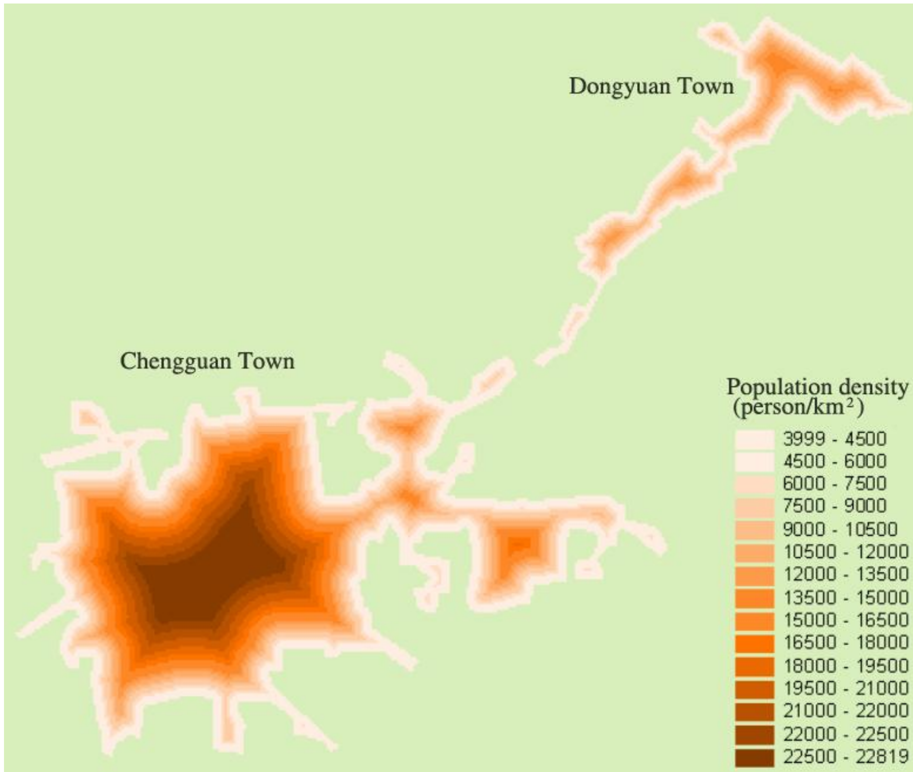

Fig. 5. Population's spatial distribution simulated by power exponential model in Ganzhou district

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