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Reflections on the activity index and related indicators

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ABSTRACT

We point out some theoretical problems in the construction of the activity index and related indicators. Concretely, if the activity index is larger than one then it is, at least theoretically, possible to decrease its value by increasing the activity in that field. Although for some practical applications these problems do not seem to have serious consequences our investigation adds to the list of problematic indicators. As the problems we point out are due to the mathematical structure of this indicator our analysis also applies to all indicators formed in the same way, such as the revealed comparative advantage index or Balassa index.

1. Introduction

The activity index (*AI*) of country *C* with respect to a given field *F* (and with respect to the world, *W*) over a given period *P* is defined as:

$$AI(C, F, W, P) = \frac{\text{the country's share in the world's publication output in the given field } F}{\text{the country's share in the world's publication output in all science fields}} \tag{1}$$

This index was introduced in informetrics by Frame (1977). It characterizes the relative research effort a country devotes to a given field *F*. It is easy to show, see Eq. (3) and e.g. (Schubert & Braun, 1986), that the activity index can also be expressed as:

$$AI(C, F, W, P) = \frac{\text{the given field's share in the country's publication output}}{\text{the given field's share in the world's publication output}} \tag{2}$$

For simplicity we will not anymore mention *W* and *P* as they are considered to be given. If a country's relative activity in a field is equal to that of the world as a whole then its $AI(C, F)$ is equal to one. From this definition it follows that a country cannot have an activity index strictly larger than one in all fields. As fields have a different weight in the total scientific production the average value of $AI(C, F)$ is, however, usually not equal to one. We recall that the $AI(C, F)$ is a version of the economists' revealed comparative advantage index or of the location quotient (Balassa, 1965). In the following sections we show that $AI(C, F)$ has some counterintuitive properties.

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2. Basic calculations involving the *AI*

We introduce the following notation:

$s(C, F)$: the publication output of country C in field F;
$t(C)$: the total publication output of country C;
$v(F)$: the total number of publications published in field F;
w: the total number of publications in the world over all fields.

Formula (2) can be written as:

$$AI(C, F) = \frac{s(C, F)/t(C)}{v(F)/w} = \frac{s(C, F) \cdot w}{t(C) \cdot v(F)} = \frac{s(C, F)/v(F)}{t(C)/w} = \text{Eq.}\,(1) \tag{3}$$

From the definition of the activity index we make the following observations.

(1) If a country is not active in a field then its corresponding *AI*-value is zero. If we consider $v(F_0)$ and w as given, then the largest value for $AI(C, F_0)$ is $w/v(F_0)$, which occurs if country C is the only country doing research in the field F_0 and if, moreover, F_0 is the only field in which C is active, so that $s(C, F_0) = t(C)$.
(2) If country C changes (increases or decreases) its output in a field F by a factor p which is exactly equal to the change in the whole field, and this in such a way that the country's and the world's total outputs do not change, then the new value of $AI(C, F) = (s(C, F) \cdot p \cdot w)/(t(C) \cdot v(F) \cdot p)$, which is equal to the old value.
(3) Similarly, if country C changes its total output by a factor q which is exactly equal to the change in the world's output (over all fields) and this in such a way that the country's output in field F and the total output in this field do not change, then the new value of $AI(C, F) = (s(C, F) \cdot w \cdot q)/(t(C) \cdot q \cdot v(F))$, which is equal to the original one.
(4) Combining these two actions in the sense that country C changes its output in the field F by a factor p which is exactly equal to the change in the whole field and it also changes its total output by a factor q which is exactly equal to the change in the world's output (over all fields) then $AI(C, F)$ stays the same. In other words: *AI* is invariant under these scaling operations (Liang & Rousseau, 2007).
(5) These are all reasonable properties, yet from $AI(C, F) = (s(C, F) \cdot w)/(t(C) \cdot v(F))$ we see that if the world's activity in other fields and other countries increases, while nothing happens in field F and in country C, then $AI(C, F)$ increases: a correct result but not immediately intuitively clear. It follows, however, from the fact that *AI* is a relative indicator.

From now on we will simply write s, t, v and w for $s(C, F)$, $t(C)$, $v(F)$ and w if it is not necessary to specify C or F. Then

$$AI = \frac{s \cdot w}{t \cdot v} \tag{4}$$

Clearly the following inequalities hold: $0 \le s \le t \le w$. As we do not consider degenerate cases we assume that actually:

$$0 < s < t < w \tag{5}$$

Similarly (not taking degenerate cases into account), we have:

$$0 < s < v < w \tag{6}$$

We note the following simple property which is used in Appendix.

Proposition. *If* $s - t - v + w = 0$ *then* $AI < 1$.

Proof. As $s - t - v + w = 0$, $w = t + v - s$. Moreover, writing $t = s + a$ and $v = s + b$ with $a, b > 0$, we have to show that: $s \cdot (t + v - s) < t \cdot v$ or: $s \cdot (s + a) + s \cdot (s + b) - s^2 < (s + a) \cdot (s + b)$. This is equivalent with: $s^2 + s \cdot a + s \cdot b < s^2 + s \cdot a + s \cdot b + a \cdot b$, which is clearly correct.

3. Influence on *AI* in the case of increased activity in one field

When $AI(C, F) = 1$ the activity index behaves as expected with respect to a change in publication activity in field F. This is shown in the following proposition.

Proposition 1. *If country C has an activity index of one, AI(C, F)* = 1, *with respect to field F, and it increases its output in field F by an amount D* > 0, *then, if all other parameters stay the same, this country's AI(C, F) becomes strictly larger than 1. Similarly, if the country decreases its output in the field F by an amount d*, 0 < *d* < *s, then this country's AI(C, F) becomes strictly smaller than 1.*

Proof. If $AI(C, F) = 1$, we may write that

$$1 = AI = \frac{s/t}{v/w} = \frac{s/t}{(k \cdot s)/(k \cdot t)} \tag{7}$$

with $k > 1$, by (5) and (6).

Table 1
An example where the activity index behaves counterintuitively.

	Before	After an increase of 10 (D = 10) articles in field F	After a decrease of 10 (d = 10) articles in field F
$s(C, F)$	190	200 (190 + 10)	180 (190–10)
$t(C)$	200	210	190
$v(F)$	200	210	190
w	400	410	390
$AI(C, F)$	1.9	1.859	1.945

Adding D publications to field F and no other changes yields:

$$AI_D = \frac{(s+D)/(t+D)}{(k \cdot s+D)/(k \cdot t+D)} \tag{8}$$

We prove that AI_D is strictly larger than 1. Indeed:

$$1 < \frac{(s+D)/(t+D)}{(k \cdot s+D)/(k \cdot t+D)} \Leftrightarrow \frac{k \cdot s+D}{k \cdot t+D} < \frac{s+D}{t+D} \tag{9}$$

Inequality (9) is equivalent with:

$$k.s.t + k.s.D + t.D + D^2 < k.s.t + k.t.D + s.D + D^2 \text{ or : } k.s.D + t.D < k.t.D + s.D \tag{10}$$

As $D > 0$, inequality (10) is equivalent with

$$k \cdot s + t < k \cdot t + s \tag{11}$$

or

$$s \cdot (k-1) < t \cdot (k-1) \tag{12}$$

Finally, as $k > 1$, inequality (12) is equivalent with $s < t$, which is true by (5). This proves the first part of this proposition. The second part can be shown in a similar way and is left to the reader.□

However, if an activity index is not equal to one then it is possible that an increase in output of a country in a particular field – while all other parameters stay the same – actually leads to a decrease in the activity index of that country for that field. We consider such behaviour to be counterintuitive, yet it may happen as shown in the following artificial example (see Table 1). Such a counterintuitive result may also happen when decreasing a country's output in a field, also see Table 1.

The example provided in Table 1 is clearly an artificial case. So, when does this problem occur? We say that the activity index satisfies the additivity property if an increase D in output of a country in a particular field – while no other changes are made to all other parameters – leads to an increase in the activity index of that country for that field. Note that this requirement is rather weak as it is formulated in absolute terms, i.e. we require that an increase leads to an increase and say nothing about the amount of this increase. Similarly we formulate a subtractivity property, which states that a decrease d in the output of a country in a particular field – while no other changes are made to all other parameters – must lead to a decrease in the activity index for that country in that field.

We first prove the following lemma about additivity (writing tv for $t \cdot v$ and similarly for other multiplications).

Lemma 1. *Putting* $X = \frac{sw(t+v)-tv(s+w)}{tv-sw}$, *we have*

(I) *If* $tv - sw > 0$, *i.e.* $AI < 1$, *then the additivity property is satisfied if and only if* $D > X$;
(II) *if* $tv - sw < 0$, *i.e.* $AI > 1$, *then the additivity property is satisfied if and only if* $D < X$.

Note that if $tv = sw$, *then* $AI = 1$, *which is the case studied in* Proposition 1.

Proof. For which $D > 0$ is $\frac{s/t}{v/w} < \frac{(s+D)/(t+D)}{(v+D)/(w+D)}$?

This requirement is the same as:

$$\frac{sw}{tv} < \frac{(s+D)(w+D)}{(t+D)(v+D)}$$

Then this inequality becomes:

$$sw(tv + (t+v)D + D^2) < tv(sw + (s+w)D + D^2)$$

or $(tv - sw) \cdot D^2 + (tv(s+w) - sw(t+v)) \cdot D > 0$ or (as $D > 0$)

$$D > \frac{sw(t+v) - tv(s+w)}{tv - sw} = X \text{ if } tv - sw > 0$$

and

$$D < \frac{sw(t+v) - tv(s+w)}{tv - sw} = X \text{ if } tv - sw < 0$$

This ends the proof of this lemma.□

We check that this requirement is not satisfied in the case of the counterexample shown in Table 1. We first note that in this example $tv - sw$ is negative, namely equal to $(200)\cdot(200) - (190)\cdot(400) = -36{,}000$ (and $AI = 1.9 > 1$). Hence the requirement we have to check is whether $D < X$. This means: is $D = 10 < [190\cdot400\cdot(200+200) - 200\cdot200\cdot(190+400)]/(40{,}000 - 76{,}000) = -6800/36 \approx -188.889$? This requirement is indeed not satisfied as the left-hand side is positive and the right-hand one is negative.

Next we prove another lemma giving a relation between X and s.

Lemma 2. *If $AI < 1$ then $X < -s < 0$. In particular, X is always negative when $AI < 1$.*

Proof. $AI < 1$ is equivalent with $tv - sw > 0$.

Now: $-X > s$

$$\Leftrightarrow -\frac{sw(t+v) - tv(s+w)}{tv - sw} > s$$
$$\Leftrightarrow -sw(t+v) + tv(s+w) > sw(tv+sw)$$
$$\Leftrightarrow tvw + s^2w > swt + swv$$

Dividing by w (>0) and writing $t = s + a$ and $v = s + b$, with $a, b > 0$ yields the equivalent expression:

$$(s+a)(s+b) + s^2 > s(s+a) + s(s+b)$$
$$\Leftrightarrow ab > 0$$

This proves the lemma.□

The previous lemmas lead to the following theorem.

Theorem 1. *If $AI \le 1$ then the additivity property is always satisfied.*

Proof. The case that $AI = 1$ has already been studied. Case I in Lemma 1 showed that if $AI < 1$, then the additivity property is satisfied if and only if $D > X$. However, in this case X is always negative (as shown by Lemma 2) so that the additivity property is always satisfied.

If $AI > 1$ then we may try to find examples not satisfying the additivity property by taking $D > \frac{sw(t+v)-tv(s+w)}{tv-sw}$.

It follows from the definitions that for a given country C, $\sum_F (t(C)\cdot v(F) - s(F,C)\cdot w) = 0$. Hence in all realistic cases a country has fields for which $AI > 1$ and fields for which $AI < 1$. Consequently, there are always fields for which, taking D large enough, the additivity requirement is not satisfied.□

Next we consider the subtractivity property. We first note that always $0 \le d \le s$.

Theorem 2.

(I) *If $tv - sw > 0$, i.e. $AI < 1$, then the subtractivity property is always satisfied.*
(II) *If $tv - sw < 0$,* i.e. *$AI > 1$, then the subtractivity property is always satisfied if $X > 0$; if $X < 0$ then it is satisfied for $d > -X$.*

Proof. For which $s \ge d > 0$ is $(s/t)/(v/w) > ((s-d)/(t-d))/((v-d)/(w-d))$?

This requirement is the same as:

$$\frac{sw}{tv} > \frac{(s-d)(w-d)}{(t-d)(v-d)}$$

Then this inequality becomes:

$$sw(tv - (t+v)d + d^2) > tv(sw - (s+w)d + d^2)$$

or

$$(tv - sw)d^2 + (sw(t+v) - tv(s+w))d < 0$$

or

$$d((tv - sw)d + (sw(t+v) - tv(s+w)) < 0$$

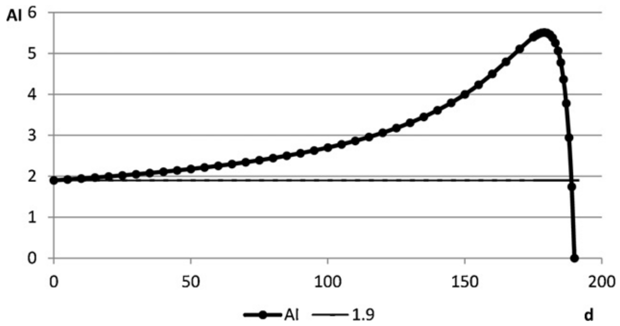

Fig. 1. Case: $s = 190$, $t = 200$, $v = 200$, $w = 400$ (as in Table 1).

or

$$(tv - sw)d\left(d + \frac{sw(t+v) - rv(s+w)}{tv - sw}\right) < 0$$

or

$$(tv - sw)d(d + X) < 0 \tag{13}$$

In a first step inequality (13) leads to four cases:

(1) $tv - sw > 0$ (or $AI < 1$) and $X > 0$;
However, this case cannot happen as we have shown in Lemma 2 that $AI < 1$ always implies that $X < 0$.

(2) $tv - sw > 0$ (or $AI < 1$) and $X < 0$;
Then the subtractivity property is satisfied for $0 < d < -X$. However, as $s < -X$ (by Lemma 2), this implies that also in this case the subtractivity property is always satisfied.

(3) $tv - sw < 0$ (or $AI > 1$) and $X > 0$;
Then the subtractivity property is always satisfied, as (13) is always negative for $d > 0$.

(4) $tv - sw < 0$ (or $AI > 1$) and $X < 0$;
Then the subtractivity property is satisfied for $d > -X$. We can find counterexamples for $0 < d < -X$.

This ends the proof of this theorem.□

Table 1 (with $d = 10$) is an example of case 4. Indeed the subtractivity property is only satisfied for $d > 188.889$ (illustrated in Fig. 1). Note that, as *AI* is equal to zero when $d = s$ the subtractivity property is always satisfied near $d = s$.

When the subtractivity property is not satisfied the *AI* reaches a maximum value, as illustrated in Fig. 1. The *d*-value for which this peak is reached is calculated in Appendix.

4. Real-world values for the activity index and *D*-values for which the additivity property is not satisfied

The previous results show that the *AI* has some flaws. Yet, how important are they? Does it matter in reality that the *AI* is not a 'perfect' measure? Are most results that make use of the activity or attractivity index invalid? As a test we calculate the smallest *D*-value for which a counterintuitive result is possible. Data are obtained from the NSI (National Science Indicators) database 2009 issued by Thomson Reuters. Field definitions are those used in the ESI (Essential Science Indicators – Thomson Reuters). Tables 2–4 show the results.

For all cases that we have checked *D* must take unrealistically high values (often more than the total world production of articles in one year) in order to violate the additivity requirement, so that in reality this does not seem to be a problem. Table 5 gives an illustration of an (unrealistic) violation of additivity based on real data for the field of chemistry in China. We found that $D = 801{,}818$ (almost seven times the total scientific production of China). Adding 800,000 ($<D$) articles still yields an increase of *AI*; adding, however, 805,000 ($>D$) articles effectively leads to a decrease of *D*.

5. Conclusions and discussions

The problem studied in this article occurs for all indicators that are constructed using the same formalism as the activity index. Such indicators have been studied, be it from a somewhat different perspective, in (Ramanana-Rahary, Zitt, & Rousseau, 2009). Examples are the attractivity index (Schubert & Braun, 1986), the Egghe–Rousseau generalizations of these indices (Egghe & Rousseau, 2002) and the document-type version of these indicators (Zhang, Rousseau, & Glänzel, 2011). It seems that the problem we pointed out has no real consequences for the interpretation of large-scale activity indices as published

Table 2
Activity index (*AI*) for some countries in 19 major disciplines, rounded to one decimal (NSI, 2009).

AI	USA	France	Germany	Japan	UK
Agricultural Sciences	0.6	0.8	0.6	0.7	0.5
Biology and Biochemistry	1.2	0.9	1.0	1.2	1.1
Chemistry	0.6	0.9	1.0	1.2	0.6
Clinical Medicine	1.2	1.0	1.1	1.0	1.2
Computer Science	1.0	1.0	0.8	0.7	0.9
Engineering	0.8	0.9	0.7	0.9	0.8
Environment/Ecology	1.1	0.9	0.8	0.5	1.0
Geosciences	1.1	1.5	1.3	0.8	1.4
Immunology	1.5	1.1	1.1	1.0	1.3
Materials Science	0.5	0.8	0.8	1.2	0.6
Mathematics	0.9	1.5	0.9	0.6	0.8
Microbiology	1.1	1.1	1.1	1.0	1.1
Molecular Biology and Genetics	1.5	1.1	1.2	1.1	1.4
Neuroscience and Behavior	1.5	0.9	1.3	1.0	1.2
Pharmacology and Toxicology	1.0	0.7	0.8	1.3	0.8
Physics	0.8	1.3	1.3	1.4	0.9
Plant and Animal Science	0.9	0.8	0.8	0.9	0.9
Psychiatry/Psychology	1.8	0.5	1.0	0.3	1.7
Space Science	1.7	2.3	2.1	1.1	2.3

AI	Belgium	Spain	Brazil	China	India	Russia
Agricultural Sciences	1.0	1.9	3.6	0.6	1.8	0.3
Biology and Biochemistry	0.9	0.9	1.0	0.8	1.0	0.6
Chemistry	0.8	1.0	0.7	1.8	1.7	1.6
Clinical Medicine	1.2	0.8	1.0	0.4	0.5	0.2
Computer Science	1.0	1.2	0.5	1.0	0.6	0.3
Engineering	0.8	1.0	0.5	1.2	1.1	0.8
Environment/Ecology	1.2	1.3	1.1	0.9	0.9	0.5
Geosciences	0.9	0.9	0.6	1.1	1.0	2.5
Immunology	1.5	0.8	0.8	0.5	0.6	0.1
Materials Science	0.7	0.7	0.6	2.1	1.6	1.3
Mathematics	0.9	1.2	0.7	1.3	0.6	2.0
Microbiology	1.2	1.2	1.2	0.7	1.4	0.6
Molecular Biology and Genetics	1.0	0.9	0.8	0.7	0.5	0.8
Neuroscience and Behavior	0.9	0.9	1.0	0.5	0.3	0.5
Pharmacology and Toxicology	1.0	0.7	1.5	0.8	1.8	0.2
Physics	0.9	1.0	0.7	1.6	1.2	2.9
Plant and Animal Science	1.2	1.2	2.6	0.6	0.9	0.4
Psychiatry/Psychology	1.3	1.0	0.5	0.1	0.1	0.2
Space Science	1.7	2.1	0.7	0.8	0.9	2.8

in the literature. Yet, we warn that there is a theoretical flaw in the construction of this type of indicators and in some circumstances it may really matter.

Because of the possible problems illustrated in this contribution a complete set of axioms characterizing indicators similar to the activity index is called for. What we have in mind is something like the Bouyssou and Marchant (2011) approach. Yet, this might prove a futile exercise as it seems that at some point all ratios and certainly ratios of ratios always behave counterintuitively. This is, for instance, shown for the journal impact factor where adding articles with zero citations may change the relative rank between two journals (Rousseau & Leydesdorff, 2011). Ratios were also one of the points discussed in the debate about the crown indicator (Larivière & Gingras, 2011; Lundberg, 2007; Opthof & Leydesdorff, 2010; Waltman, van Eck, van Leeuwen, Visser, & van Raan, 2011). We would like to recall that as shown in (Hu & Rousseau, 2009) relative indicators such as the activity index (*AI*) and the attractivity index (*AAI*) reflect relative positions with respect to reference standards, but they cannot reflect real research visibility in a given field. Moreover, Hu and Rousseau (2009) claim that these indices should not be used for interdisciplinary comparisons.

This study can be considered another addition to the line of inquiry which takes a mathematical, often axiomatic, look at scientometric indicators. As such we follow Lundberg (2007), Opthof and Leydesdorff (2010), Waltman and van Eck (2009), Bouyssou and Marchant (2011) and others. Yet, one may wonder if it is reasonable to expect that an index which is built up from ratios, such as the activity index, should be used if one expects it to have properties related to sums. Wouldn't it be reasonable to, either use such an index in a context where only ratio-type properties are required, or not to use it at all? In our view, scientometricians still have to come to terms with this question.

Finally, we mentioned that the *AI* is a version of the Balassa index or index of comparative advantage as used in trade theory, or the location quotient as used in spatial economics. Also in these fields one has found several shortcomings of these

Table 3
Smallest *D*-value (this is *X*) leading to a violation of the additivity property (only cases where $AI > 1$), based on the data shown in Table 2.

Smallest *D*-value (*X*)	USA	France	Germany	Japan	UK
Agricultural Sciences					
Biology and Biochemistry	3,625,657			3,729,453	9,628,968
Chemistry				5,536,071	
Clinical Medicine	2,464,807		13,427,394		3,716,632
Computer Science	26,091,887	38,265,449			
Engineering					
Environment/Ecology	9,089,473				27,939,945
Geosciences	6,510,226	1,936,158	3,926,363		2,395,688
Immunology	1,228,754	10,781,674	7,432,852		2,958,290
Materials Science				4,617,112	
Mathematics		1,933,588			
Microbiology	5,334,732	6,747,838	19,246,855	62,291,114	12,981,826
Molecular Biology and Genetics	1,116,643	11,822,390	4,683,351	11,435,009	2,254,133
Neuroscience and Behavior	1,232,572		3,557,477		3,841,625
Pharmacology and Toxicology				3,174,310	
Physics		3,030,799	3,330,652	2,292,968	
Plant and Animal Science					
Psychiatry/Psychology	694,196		130,615,659		1,227,081
Space Science	779,346	674,600	913,238	9,321,438	696,301

Smallest *D*-value (*X*)	Belgium	Spain	Brazil	China	India	Russia
Agricultural Sciences	63,877,802	1,045,497	333,859		1,216,490	
Biology and Biochemistry			27,914,923		22,922,463	
Chemistry		31,029,008		801,818	1,114,558	1,297,686
Clinical Medicine	4,056,191					
Computer Science		4,363,901		48,945,559		
Engineering		32,034,903		3378,025	7,552,164	
Environment/Ecology	6,202,502	3,079,804	9,500,560			
Geosciences				18,056,592		600,839
Immunology	1,965,579					
Materials Science				687,278	1,638,459	3,441,542
Mathematics		6,406,401		2,721,343		911,749
Microbiology	4,580,964	5,179,395	4,678,565		2,234,368	
Molecular Biology and Genetics	20,952,876					
Neuroscience and Behavior			36,299,833			
Pharmacology and Toxicology	175,985,203		2,211,362		1,208,855	
Physics				1,169,123	4,403,207	378,532
Plant and Animal Science	4,214,137	3,995,728	531,879			
Psychiatry/Psychology	3,465,939					
Space Science	1,541,501	866,693				548,092

Table 4
Smallest *D*-value in some subfields for which a violation of the additivity property can be obtained; data retrieved from NSI, 2009.

Country	WoS field	*D*-value necessary to obtain a violation of the additivity property
USA	Substance abuse	296,749
China	Crystallography	601,270
Belgium	Microscopy	1,491,260
Bulgaria	Multidisciplinary sciences	222,000

Table 5
An illustration of an (unrealistic) violation of additivity based on real data.

	Real data China – chemistry	Adding $D = 800{,}000$ articles to chemistry	Adding $D = 805{,}000$ articles to chemistry
s	26,114	826,114	831,114
t	116,150	916,150	921,150
v	134,077	934,077	939,077
w	1,083,475	1,883,475	1,888,475
AI	1.81685	1.81824	1.81443

Table 6
Examples: underlined are cases where the subtractivity property is not satisfied and a peak for AI_d occurs. Extrema outside the interval [0,s] have no practical meaning.

s	t	v	w	AI	X	Extrema occur for d =	Notes
190	200	200	400	1.9	−188.89	<u>178.95</u> and 200	Example presented in Table 1 and Fig. 1
190	240	210	450	1.696	−177.18	<u>147.37</u> and 222.1	$AI > 1$ and $0 < -X < s$
50	100	150	200	0.667	−250	94.44	Example where $s-t-v+w=0$; $AI < 1$
190	300	600	2000	2.111	261	−98.9 and 409.01	$AI > 1$ but $X > 0$
150	200	210	500	1.786	−104.55	<u>70.185</u> and 204.81	$AI > 1$ and $0 < -X < s$

measures and alternatives have been proposed (see e.g., Hoen & Oosterhaven, 2006; Vollrath, 1991). Yet, most alternatives focus on using other factors and not on restructuring the formula itself. Only Hoen and Oosterhaven's (2006) aggregated additive revealed comparative advantage index (aggregated ARCA) restructures the formula. Translated into the language of scientometrics their aggregated *ARCA* is:

$$ARCA(C) = \frac{1}{2}\sum_F \left| \frac{s(C,F)}{t(C)} - \frac{v(F)}{w} \right| \tag{14}$$

Eq. (14) is an aggregate index (a sum over all fields) for a country with values between 0 and 1. It is related to a sum of Gini indices. If the relative contribution of field F in the total number of publications of country is equal to the relative contribution of field F in the world output then the term corresponding to field F is zero. If this happens for all fields then $ARCA(C) = 0$. If country C is only active in a specific field F_0, and it is moreover, the only country active in this field, then $ARCA(C)$ is close to 1.

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Appendix.

We assume that $AI \neq 1$. Writing x for d (in order to distinguish between d and the symbol for derivation), we have

$$\frac{d}{dx}AI_d(x) = \frac{d}{dx}\left(\frac{(s-x)(w-x)}{(t-x)(v-x)}\right) = \frac{(s-t-v+w)x^2 - 2x(sw-tv) + (stw - stv + svw - tvw)}{(t-x)^2(v-x)^2}$$

Assuming that $s-t-v+w \neq 0$, this expression is zero for

$$x = \frac{-(\sqrt{(s^2 - s(t+v) + tv)(t-w)(v-w)} - sw + tv}{s-t-v+w} \text{ and } x = \frac{-(\sqrt{(s^2 - s(t+v) + tv)(t-w)(v-w)} + sw - tv}{s-t-v+w}$$

Otherwise, if $s-t-v+w=0$, $x = (stw - stv + svw - tvw)/2(sw - tv)$. Yet, when $s-t-v+w=0$ then $AI < 1$ and hence the subtractivity property is always satisfied. Table 6 gives some examples.

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